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HEAT TRANSFER IN SHAPED CHANNELS IN TRANSPIRATION COOLING

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The combined effect of a change in the coolant flow rate and account for heat transfer before entry into a shaped channel on the enhancement of the thermal mode in transpiration is analyzed.

In theoretical investigations of phenomena of heat transfer in transpiration cooling, difficulties arise at the level of both mathematical and physical modeling. The main problem lies in the arbitrariness of the structure of the porous material where the boundary between two phases is a set of many discrete portions with different curvature. It is precisely this fact that not only makes theoretical investigation of interphase heat transfer difficult but also hinders universal, in the sense of methodology, experimental works in these media. By virtue of this, a research approach with the use of simplified models based on the assumption of regularity of the porous-material structure or with the use of so-called "tubular" models based on equivalent transition from the real porous material to a perforated region is quite justified. Here it should be noted that heat-transfer processes have, as a rule, been studied in the context of general laws of conservation of mass, momentum, and energy. However, situations are rather common where the suggested systems turn out to be incorrect due to uncertainty in the conditions of closure or due to the absence of a unique methodological approach to the search for the parameters responsible for the closure. The question is the determination of the volumetric coefficient of heat transfer and the formulation of the conditions of heat transfer at the entry into the porous material [1].

Thus, in [2, 3] a version of the determination of both the volumetric and surface coefficients of heat transfer at the entry into a porous material was suggested. A dimensionless relation for the volumetric Nusselt number $\mathrm{Nu_v}$ is represented as

$$\mathrm{Nu_v} = 2P\,\mathrm{Nu}$$

and includes the porosity P of the material and the surface intrapore Nusselt number Nu, where the radius of a model capillary is taken as the characteristic dimension. To close the boundary conditions at the entry into the porous material we drew on the analogy to heat transfer in flow past spherical, cylindrical, and flat particles. A result of the analysis was recommendations on calculation of the Stanton number

$$\mathrm{St} = \frac{A}{\mathrm{Re}^{0.5}\,\mathrm{Pr}^{0.6}}\,, \qquad (1)$$

where $A = 1.315$ for a sphere, $A = 1.14$ for a cylinder, and $A = 0.57/\sqrt{2\pi}$ for a normally arranged plate.

At the same time, the development and subsequent introduction of systems of transpiration cooling assume the acceptance of more exacting demands on the parameters responsible for both the improvement of the efficiency of the heat-protection properties of the system and the possibilities of control over the thermal processes. In studying these problems, we note the developed approaches to the enhancement of heat transfer.

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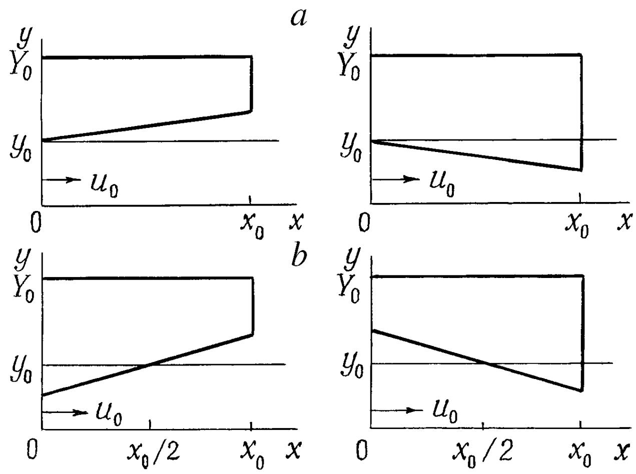

Fig. 1. Coordinate system in transpiration cooling in a shaped channel for constant (a) and variable (b) cross sections at the entry.

These are methods of disturbance of the liquid flow due to local turbulization, installation of fins, change of the velocity of the flow in the channels, and, finally, design of shaped channels capable of providing longitudinal isothermicity of the heat-transfer surface. In the latter case, as a rule, the search for original techniques of construction of channels with a variable cross section was based on the quest for optimization of intrachannel heat transfer. Here, in most research works the process of heat transfer before entry into the porous material either was not considered or did not receive proper attention. Moreover, the construction of an optimum geometry for the channel was rather often accompanied by a change in the area of the inlet cross section and, as a consequence, by a substantial correction of such an important characteristic of convective heat transfer as the coolant flow rate. Therefore, combined account for thermal phenomena and analysis of characteristics affecting the process of transpiration cooling will allow one not only to show the special features of this system of heat protection but also to refine the main parameters of control. Therein lies the importance and urgency of more complete physicomathematical models.

In the work proposed, we give results of determination of the contribution of the heat-transfer processes in the region before entry into a flat shaped model channel (convergent, divergent) to the total thermal mode. The effect of a change in the coolant flow rate on the efficiency of transpiration cooling is analyzed.

Linearly converging or diverging walls of a long channel are considered. Here, the vertex angles of the divergent channel are such that there is no separation of the boundary layer. According to [4], the admissible semivertex angle is 4.8^{o}. Figure 1 presents computation regions of different shaped channels. The thermal mode is studied for both a constant flow rate at the entry (Fig. 1a) and a variable flow rate that depends on the channel geometry (Fig. 1b). We note that the change in the flow rate at the entry is related to the angle of rotation of the channel generatrix relative to the point ($x_0/2$, y_0). Moreover, the Nusselt number is taken as the observed parameter, and the semivertex angle of the converging or diverging channels as the governing parameter. Natural limitations on the value of the angle of rotation are formulated proceeding from the following conditions: nonclosure of the inlet and outlet cross sections, finite thickness of the channel wall, and nonseparating flow. To simplify the computation regions, we introduce the following variables: $\xi = x/x_0$ and $\eta = y/y_0 f$ for the channel, $\eta = (y - y_0 f)/(Y_0 - y_0 f)$ for the channel wall. Mathematical modeling is carried out on the basis of approximation of a narrow channel for laminar flow of a thermally compressible viscous gas [5], and in dimensionless form, as applied to shaped channels, it is transformed to the form

$$\frac{\partial (f\rho)}{\partial \tau} + \frac{\partial (f\rho u)}{\partial \xi} + \frac{\partial (f\rho w)}{\partial \eta} = 0 , \qquad (2)$$

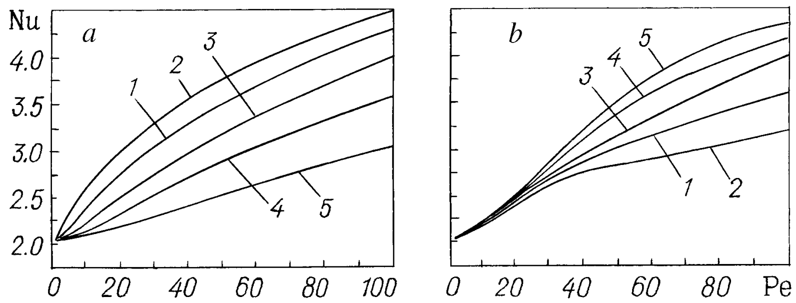

Fig. 2. Nu number vs. Pe number for constant (a) and variable (b) inlet cross sections of the channel: 1, 2) diverging channel (the semivertex angle is $\gamma = 1$ and 2°, respectively); 3) ordinary flat channel; 4, 5) converging channel ($\gamma = -1$ and -2°).

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial \xi} + w\frac{\partial u}{\partial \eta} = -\frac{1}{\rho}\frac{\partial P_{\mathrm{d}}}{\partial \xi} + \frac{1}{\mathrm{Re}\,\delta f^2}\frac{\partial}{\partial \eta}\left(\mu\frac{\partial u}{\partial \eta}\right), \tag{3}$$

$$\frac{\partial P_{\mathrm{d}}}{\partial \eta} = 0\,, \tag{4}$$

$$\rho T = 1\,, \tag{5}$$

$$\frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial \xi} + w\frac{\partial T}{\partial \eta} = \frac{1}{\mathrm{Pe}\,\delta \rho f^2}\frac{\partial}{\partial \eta}\left(\lambda\frac{\partial T}{\partial \eta}\right), \tag{6}$$

$$\frac{\partial T_1}{\partial \tau} = \frac{\delta}{\mathrm{Pe\,Ka}}\frac{\partial^2 T_1}{\partial \xi^2} + \frac{1 - k^2\,(1-\eta)^2}{\mathrm{Pe\,Ka}\,\delta\left(\dfrac{Y_0}{y_0} - f\right)^2}\frac{\partial^2 T_1}{\partial \eta^2}\,, \tag{7}$$

where $w = (v - u\eta f')/f$, $f = 1 - \frac{k}{2\delta} + \frac{k}{\delta}\xi$ for Fig. 1b, $f = 1 + \frac{k}{\delta}\xi$ for Fig. 1a, f' is the derivative with respect to ξ. The variability of the thermophysical characteristics is given as $\mu = T^n$ and $\lambda = T^m$.

The boundary conditions determining the process of transpiration cooling are written with account for heat transfer before entry into the channel:

$$\tau = 0: \quad u = u_0\,(\xi, \eta)\,, \quad w = w_0\,(\xi, \eta)\,, \quad T = 1\,, \quad T_1 = T_{10}\,;$$

$$\xi = 0: \quad u = 1\,, \quad \frac{\partial T_1}{\partial \xi} = \mathrm{Bi}\,(T_1 - 1)\,, \quad \mathrm{St} = \frac{T - 1}{T_1 - 1}\,;$$

$$\xi = 1: \quad T_1 = T_{10}\,;$$

$$\eta = 0: \quad \frac{\partial T}{\partial \eta} = \frac{\partial u}{\partial \eta} = 0\,, \quad w = 0\,;$$

$$\eta = 1: \quad u = w = 0, \quad T = T_1, \quad \frac{\partial T}{\partial \eta} = K_\lambda \frac{\partial T_1}{\partial \eta}, \quad K_\lambda = \frac{\lambda_1}{\lambda}.$$

The condition of heat insulation is formulated on the outer surface of the channel wall. It should be noted that the formulation of interrelated boundary conditions of the third kind for T and T_1 in the inlet cross section reflects the mechanism of thermal interaction between the injected gas and the end part of the channel wall [1]. The unknown parameter that arises here, in the form of the Stanton number, is determined, as suggested in [2], from relationship (1), which corresponds to flow past a normally arranged plate. As a consequence, the number Bi is calculated from the formula

$$\mathrm{Bi} = \frac{\mathrm{St\ Pe}}{\delta K_\lambda},$$

where the half-width of the channel is taken as the characteristic dimension.

Thus, the above-formulated mathematical model is a conjugate problem of nonstationary convective heat transfer in a shaped flat channel with account for the thermal interaction between the gas and the wall before entry into the channel.

The problem is solved numerically based on the iteration-interpolation method [6], with consistency of the fields of velocity and pressure being reached by simultaneous determination of them [7] using a modification of the method for the nonstationary case [8]. A computational grid with a number of nodes 51 × 19 that is uniform along the flow and has bunching of the nodes in the near-wall region is used in the channel. A 51 × 11 uniform grid is taken in the channel wall. The stationary solution of the system of equations (2)–(4) in isothermal gas flow is taken as the initial distribution of the components of the velocity vector.

The solution of the system of difference equations was tested for both the downstream region in the zone of thermal and hydrodynamic stabilization and the initial thermohydrodynamic portion. A comparative analysis with monograph [9] behind the portion of link-up of the thermal and hydrodynamic boundary layers for a constant temperature and heat flux on the wall showed that the relative error for the Nusselt number does not exceed 0.25%. The efficiency of the numerical algorithm was also tested in the inlet region of the channel. Check calculations were done on the initial thermal portion with a fully developed profile of velocity and a constant heat flux on the wall and also for the general case [10] – the initial thermohydrodynamic portion. The calculated relative error for the Nusselt numbers did not exceed 1.7% with variation of the Péclet numbers from 2 to 150. We note that downstream and also with increase in the Péclet number the error of the computational algorithm decreases and tends to values that correspond to stabilized heat transfer.

The results of the comparative analysis allow one to speak of a sufficient capacity of the employed mathematical model to solve the problem and the possibility of carrying out numerical calculations for more complex thermal phenomena.

We consider the process of convective cooling of the wall of a shaped flat channel within the range of variation of the Péclet number from 2 to 100 for Pr = 0.7, $\delta = 0.05$, $Y_0/y_0 = 3$, Ka = 1.84, $K_\lambda = 1770$, $n = m = 0.76$, and $T_{10} = 2$ and with account for the heat exchange between gas and the wall before entry into the channel. The indicated limitations on the value of the angle of slope of the channel generatrix lead in this case to the conditions: $|k| < \delta$ for Fig. 1a and $|k| < 2\delta$ for Fig. 1b. Specific calculations were done for the range $|k| \le 0.0349$, which corresponds to angles from -2 to $+2^{\circ}$.

Results on the intensity of heat transfer in converging and diverging channels as a function of the Péclet number and the semivertex angle with a constant inlet cross section are presented in Fig. 2a. Curve 3, which determines the thermal mode in a straight flat channel, is taken as a reference. Comparison of the data obtained indicates the advantage of diverging channels (curves 1, 2), which manifests itself over the entire considered range of Péclet numbers. The efficiency improves with increase in the angle of the channel generatrix (the relative buildup amounts to 9.5% with increase in the angle from 1^o to 2^o). We note that, compared to the straight channel, a maximum is observed that amounts to 19% at a Péclet number equal to 20. Curves 4 and 5 for converging channels reflect deterioration of the heat-transfer intensity compared not only to the diverging regions but also to the straight channel. The tendency for a decrease is enhanced with increase in both the angle of the generatrix and the Péclet number. The value of the relative decrease in the Nusselt number reaches 24%.

A qualitatively different mechanism of heat transfer is realized in the case (Fig. 1b) where the gas flow rate at the entry changes and depends completely on the angle of slope of the channel generatrix. We note that in this case the areas of the heat transfer surface that are the end parts of the channel wall on the sides of both the inlet and the outlet change automatically. In Fig. 2b, curves 1 and 2 corresponding to diverging channels turn out to be lower, in contrast to Fig. 2a, than curve 3. Here, with increase in the angle of slope from 1^o to 2^o the Nusselt number decreased by a relative value of 12%, and by 19.5% from the reference curve. Curves 4 and 5, which characterize, as previously, converging regions, also changed their position and, conversely, turned out to be higher than curve 3. The relative changes in the Nusselt number amounted to 3 and 11%, respectively. This sharp change in the character of the heat transfer is explained, first of all, by a variable dependence of the gas flow rate (a range of 0.64–1.3) on the angle of slope of the generatrix. A substantial difference in the curves in Fig. 2a and b for values of the Péclet number up to 40 is a characteristic feature revealed in the calculations. Whereas in Fig. 2a enhancement and deterioration of heat transfer manifest themselves immediately in the entire range of Péclet numbers, in the second case these manifestations are insignificant.

Account for the heat exchange between the channel wall and the cooling gas before entry into the shaped channel in all the calculations done manifests itself mainly in the initial stage of nonstationary heat transfer and to a larger extent for small Péclet numbers. Relative preheating of the gas can reach 10% and postpone the onset of the mode of quasistationary heat transfer by 35%.

Thus, the data obtained indicate high sensitivity of the process of enhancement of heat transfer not only to the degree of shaping of transport channels (the angle of slope) but also to a change in the gas flow rate. Thus, when the flow rate at the entry is constant and equal to the flow rate in an ordinary straight channel, diverging transport regions can be used for enhancement of heat transfer. If shaped channels that change the flow rate compared to a straight channel are used, then one should select converging regions.

The theoretical modeling conducted makes it possible not only to estimate the effect of the determining parameters of the system on the thermal mode in shaped channels, but also to optimize them in the context of control over transpiration cooling. In modeling the thermal mode in porous materials in the context of an equivalent transition to perforated regions, it becomes possible to evaluate the volumetric Nusselt number. The results of the paper can be used in calculation of the geometry of transport channels and in numerical experiments on evaluating the thermal mode in porous materials, including the region before entry into them.

NOTATION

$\tau = tu_0/x_0$, ξ, and η, dimensionless time and dimensionless longitudinal and transverse coordinates; $u = \dot{u}/u_0$, $v = \dot{v}/v_0$ $(v_0 - u_0 y_0/x_0)$, $T = \dot{T}/T_0$, and $T_1 = \dot{T}_1/T_0$, relative longitudinal and transverse velocities of the gas and temperatures of the gas and the wall; y_0 and x_0, half-height and length of the channel; u_0, mean velocity at the entry; $P_d = P/\rho_0 u_0^2$, relative dynamic pressure; $\rho = \dot{\rho}/\rho_0$, relative density of the gas; a and a_1, coefficients of thermal diffusivity of the gas and the wall; λ and μ, dimensionless coefficients of thermal conductivity of the gas and dynamic viscosity; $\mathrm{Ka} = a/a_1$; $\delta = y_0/x_0$; k, tangent of the angle of slope of the inner wall of the shaped channel; Re, Pe, and Pr, Reynolds, Péclet, and Prandtl numbers.

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